PHILOSOPHICAL TRANSACTIONS.

I. *Discussion of Tide Observations at Bristol.* *By* T. G. BUNT, *Bristol.*
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THIS paper contains the result of an attempt to discover empirically the Laws of the Diurnal Inequality of the Times and Heights, and of the Solar Inequality of the Times of High Water at the Port of Bristol.

The observations employed in this discussion are those that have been taken by the Bristol Self-registering Tide-Gauge, which has been kept steadily at work, with a few occasional interruptions, from the period of its erection in 1837 to the present time. This instrument consists essentially of a Clock, a Cylinder, a Float, and a Pencil, by means of which every tide marks a curve on a sheet of paper, from which the time and height of high water are ascertained. Its details are described, and an engraving of it given, both in the Philosophical Transactions for 1838, Part II., p. 249, and in the Article "Tides and Waves" in the Encyclopædia Metropolitana, written by the present Astronomer Royal.

During the far greater part of these twenty-nine years, the Tide-Gauge has been at work under my own continual inspection; and it may be important to remark that the clock has, from the commencement, been carefully adjusted to Bristol mean time by transit observations: viz. during the first fourteen years by comparison with the Transit Clock of Messrs. MUSTON and GATH, chronometer makers in this city, besides frequent sextant altitudes of the sun taken by myself, and for the last fifteen years by a transit instrument in my own house.

Soon after the Meeting in 1836 of the British Association in Bristol, I was employed under its auspices, in assisting the late Dr. WHEWELL in his discussions of the Bristol Tides; the results of which appeared in a succession of papers in the Philosophical Transactions, extending from 1837 to 1840.

The Diurnal Inequality of the Tides was that branch of the subject to which Dr. WHEWELL'S attention was specially directed. In explanation of this term, if any such be necessary, it may be stated briefly that successive tides do not increase or decrease by a regular progression, but are alternately higher and lower than such a progression would require. The intervals also between the times of high water and the preceding lunar transit to which they are referred, are found to be alternately longer and shorter, so as to present a similar irregularity.

The Diurnal Inequality of Height in the Tides of Bristol, though of no great magnitude, averaging only $2\frac{1}{2}$ inches up and down, is at once apparent on the most cursory examination; the inequality of time is not quite so easily detected. It was indeed stated by Sir JOHN LUBBOCK so lately as in 1839, in his 'Elementary Treatise on the Tides,' p. 39, that "the diurnal inequality in the interval is inappreciable on our coasts;" and again, at p. 40, "the diurnal inequality in the time of high water on our coasts is too minute to be detached from the inevitable errors of observation."

There will, I apprehend, be little difficulty in showing that this remark of Sir JOHN LUBBOCK'S is not applicable to the port of Bristol. For although the diurnal inequality in the times of high water is not large, its average magnitude, on comparing two successive intervals, being little more than four minutes between both, or two minutes for the single inequality in each, yet it is almost always perceptible in the Tide-Gauge Observations in tolerably calm weather. In the accompanying Plates of six months' observations in 1865, the diurnal inequality of both times and heights is everywhere apparent.

At Dr. WHEWELL'S request, I bestowed much time and labour from the first, in endeavouring to trace out the laws of both these inequalities, and especially that of the times, but for several years with but little success, chiefly for want of a greater number of observations. After several modes of arrangement which Dr. WHEWELL suggested had been tried, he at length arrived at the conclusion that, in order to succeed, we must divide our observations, first into twenty-four groups for the twenty-four half months, and then each of these into twenty-four smaller groups for the twenty-four hours of lunar transit. It thus became evident that little progress could be made in this investigation, until a large mass of observations had been accumulated.

These being now obtained, I have again taken the subject in hand, and embodied the results in the accompanying Plates. For the Inequality of the Times I have taken nearly the whole of the observations. The anterior epoch employed throughout is that of the third preceding lunar transit, averaging an interval of about forty-four hours. The time of every high water had been already calculated (for my annual tide table) by adding to the time of moon's transit the corresponding semimenstrual interval, with the corrections for lunar and solar parallax and declination. From this computed time, the observed time of high water had been subtracted, and the residue, or error, with its proper sign, recorded; being supposed to consist mainly of the uncorrected diurnal inequality. The amount of the inequality was taken thus. Let a be the error of a time of high water computed from a *south* transit of the moon, and b, b' the preceding and following error, then $(a-b)+(a-b')$ will be $=4$ times the inequality of the high water which has the error a. The quadruple inequality of every *south* transit observation was thus taken, and inserted in its proper group. The $(24\times24=)576$ groups thus formed being completed were then cast up, averaged, and divided by 4: the quotients were the terms out of which the fig 1, Plate I. was constructed.

Fig. 2, the diurnal inequality of height, was obtained in a precisely similar manner, except that only about fourteen years' observations were employed in its construction.

The tide to which both these figures refer, is that which follows the *south* transit at an interval of about forty-four hours. In each figure the twenty-four vertical lines represent the twenty-four hours of this south transit, *apparent* time. The twenty-four horizontal lines represent the half months, and are the axes from which the curves are measured, upwards and downwards, the scale being $\frac{1}{15}$ of an inch to a minute of time in fig. 1, and $\frac{1}{15}$ of an inch to an inch of height in fig. 2. When the curve is above the axis, it indicates in fig. 1 that the time is *later*, in consequence of the inequality; and in fig. 2 that the height is *greater* than it would have otherwise been, and *vice versâ*. The distances between these horizontal lines, or axes, are arbitrary, being merely a matter of convenience.

Both these sets of curves were laid down exactly according to the averages of the groups, without any arbitrary alteration whatever. In forming the groups, a very few enormous residues, evidently the effects of storms, or other accidental causes, were excluded. Their number was, I believe, much less than 1 per cent. Of a still less number of residues, one half of their amount only was taken, instead of the whole.

My next attempt was to improve the solar inequality correction, or that which is due to the variations of the solar parallax and declination. This was a more laborious affair, requiring the recalculation of the 19,000 observed times of high water, in order that the same formulæ, or curves of lunar correction might be employed throughout. The residues found after these new calculations were arranged, as those for the diurnal inequality had been, for each half month, and each of the twenty-four hours of transit; as I wanted to see whether anything would be gained by keeping the observations taken during the first half lunation separate from those taken during the second. I have never made this separation in arrangements for finding or improving any of the lunar curves, nor ever until now in those for the solar inequality; nor am I aware that it has ever been done by any other person. These results are contained in Plate II. fig. 3.

For the purpose of more easily comparing together these two portions of the curves, I have dotted in a copy of the curves lying between the hours $12^h\ 54^m$ and $0^h\ 54^m$ upon those of the first twelve hours of transit. The difference in many places is considerable, and evidently systematic.

The circumstance which caused the average of each group to fall on the fifty-fourth minute of the hour, was, that the residues were not posted singly, but in pairs, the error of the term computed from a *south* transit of the moon being in every case combined with the following one.

In order to separate, at least approximately, the effects of the solar parallax and declination from each other, the following method was adopted. The curves of June 8th and June 23rd were combined in one pair, and those of December the 8th and 23rd in another. Their mean parallaxes and declinations, according to the Nautical Almanac for 1866, became as follows:—

	☉'s Decl.	☉'s H. Par.
	° ′	″
June 8 . . .	22 51	8·45
June 23 . . .	23 26	8·44
Mean . .	23 8	8·445
Dec. 8 . . .	22 44	8·71
Dec. 23 . . .	23 27	8·72
Mean . .	23 5	8·715

Here we have a difference of 0″·27 of parallax, with only 0° 3′ of declination.

From these pairs of curves are obtained the solar parallax curves in Plate II. fig. 4.

In like manner, by combining eight curves in two sets of four curves in each, we obtain the following means:—

	☉'s Decl.	☉'s H. Par.
	° ′	″
March 23 . .	1 34	8·60
April 8 . .	7 14	8·56
Sept. 23 . .	0 5	8·555
Oct. 8 . .	5 54	8·59
Mean (from squares of the declinations)	4 40	8·576
June 8 . . .	22 51	8·45
June 23 . . .	23 26	8·44
Dec. 8 . . .	22 44	8·71
Dec. 23 . .	23 27	8·72
Mean . .	23 7	8·58

Here the mean declinations are 4° 40′, 23° 7′; while the parallaxes differ only 0″·004.

These give the solar declination curves in Plate II. fig. 5.

During the remarkably fine weather of the summer of 1865, I found the curves drawn by the pencil of the tide-gauge on the sheet of paper wrapped round the cylinder, more symmetrical and regular, and the agreement of the registered times of high water with those predicted in my tide-table closer than I had ever known them before. The mean error of the predicted times (found, not algebraically, by taking the balance of those + and those −, but by adding all the magnitudes together, regardless of signs) was from the 10th of April to the 24th of October only 2½ minutes; and during about six weeks, namely, from the 17th of August to the 27th of September, it was less than 1·9 minute per tide.

The diagram No. 4 shows the whole of these six months' times and heights of high water, both as predicted and registered; and is interesting chiefly because it so clearly exhibits the two Diurnal Inequalities. No one who looks at it can fail to detect, in an instant, the existence of them both; and the one is as manifest as the other. The agreement everywhere seen between the computed and observed Diurnal Inequalities, the laws of which it has cost me so much labour to attain, has, I confess, afforded me no small gratification.

Accompanying this diagram, I have enclosed a sheet taken from the Cylinder of the Tide-Gauge, containing the original markings of the pencil, on which an ink line has been very carefully drawn. It registered some of the tides of the remarkably tranquil period which has been already referred to, and is sent as a specimen of the great regularity which the curves sometimes exhibit*.

Postscript.

Received October 27, 1866.

Barometric Inequality.

In a letter of mine inserted in the Report of the British Association in 1841, I stated that from a comparison of three or four years' computed and observed Heights of High Water at Bristol, I had found that a fall of 1 inch in the mercurial column was accompanied by an average rise of about $13\frac{1}{2}$ inches of tide. I have since obtained for fourteen additional years the following proportions of tide and mercury:—

Year	Tide. in.	
1841	13·0	to 1 inch of mercury.
1842	11·4	
1843	13·2	
1844	11·4	
1845	10·6	
1846	14·7	
1847	16·0	
1848	13·7	
1849	10·0	
1850	9·5	
1851	11·0	
1852	11·7	
1853	12·0	
1854	12·0	
	14)170·2	
Mean	12·157	inches of tide to 1 inch of mercury.

The mean of all the twenty-one years I have thus examined, viz. 1834 to 1854, is 12·772 inches of tide to 1 inch of mercury.

* It has not been deemed necessary to give a Plate of this Tide-Gauge Sheet, or of Diagram No. 4. The latter is, however, preserved in the Archives of the Society, accompanied by a detailed explanation.

Postscript.

On comparing, *inter se,* the curves of the diurnal inequality of time, a very close resemblance is found between those which differ 6 months in respect of date, and 12 hours in respect of transit. For example, the curve for April 28th, 0^h, 6^h, 12^h, 18^h, 0^h, is almost identical with that of October 28th, 12^h, 18^h, 0^h, 6^h, 12^h.

The curves of the diurnal inequality of height present a good deal of the same resemblance.—*April* 1867.

Diurnal Inequality.

Fig. 1.
TIME.

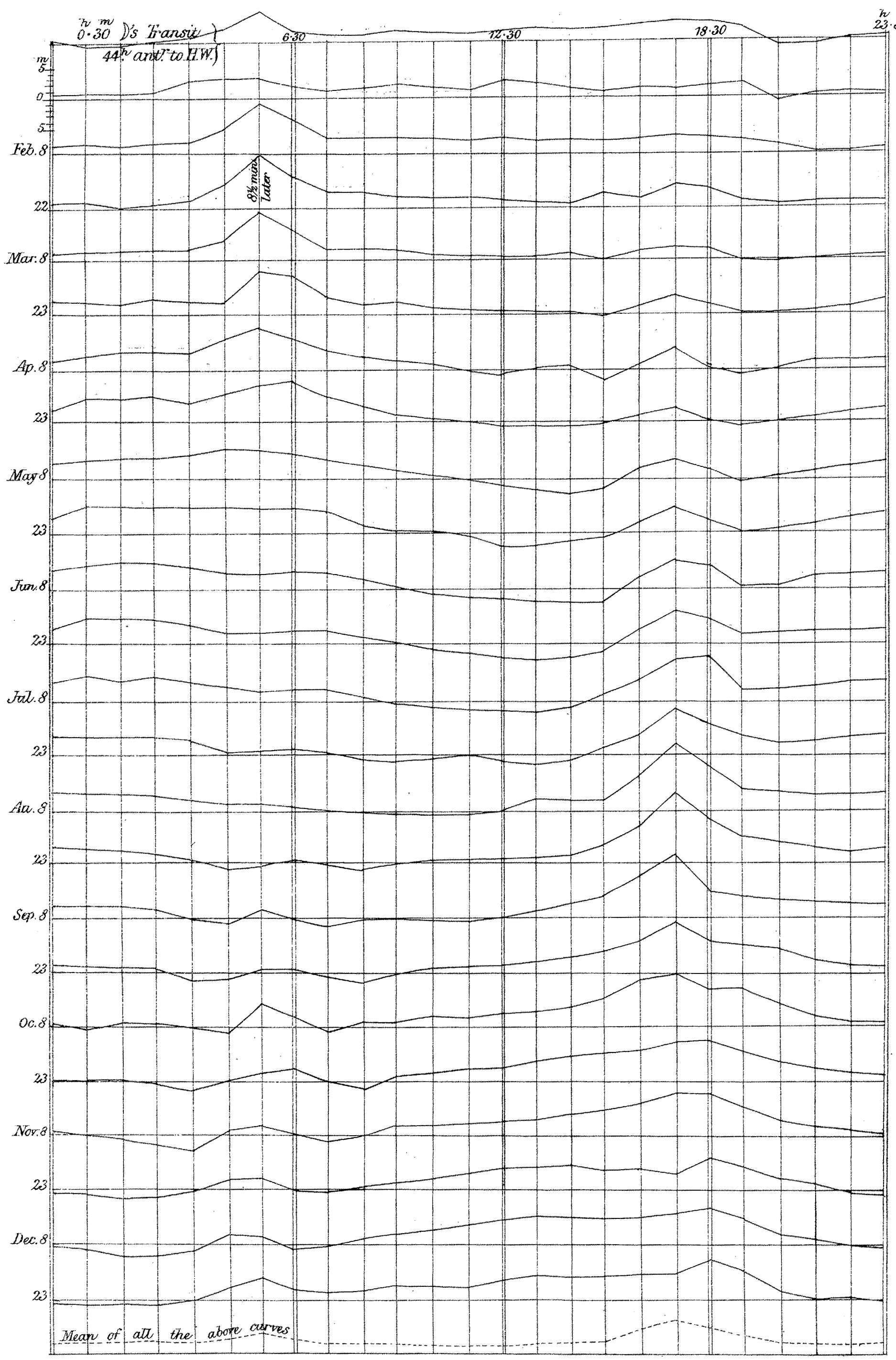

Fig. 2.
HEIGHT

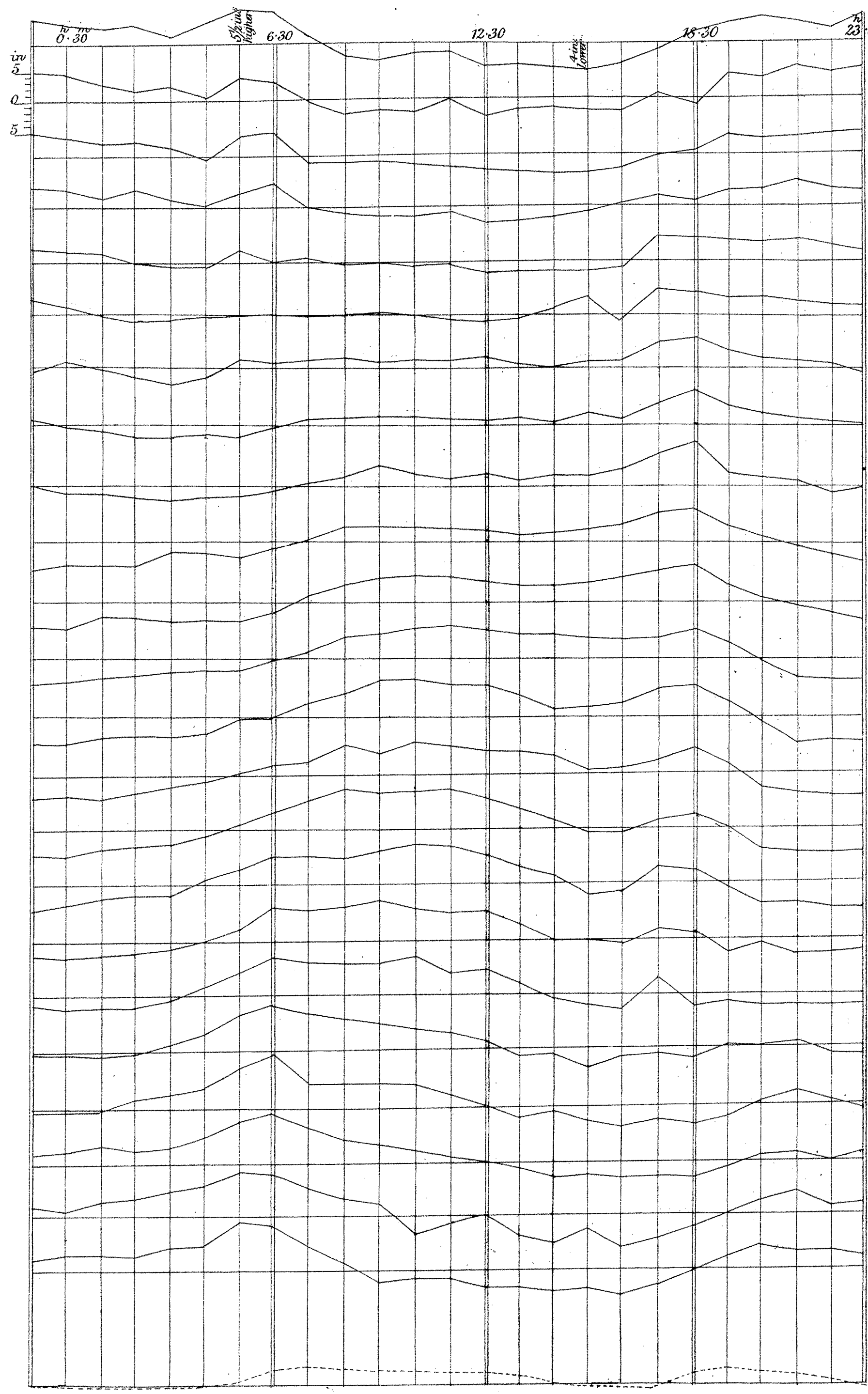

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Fig. 3.

Solar Inequality.

TIME.

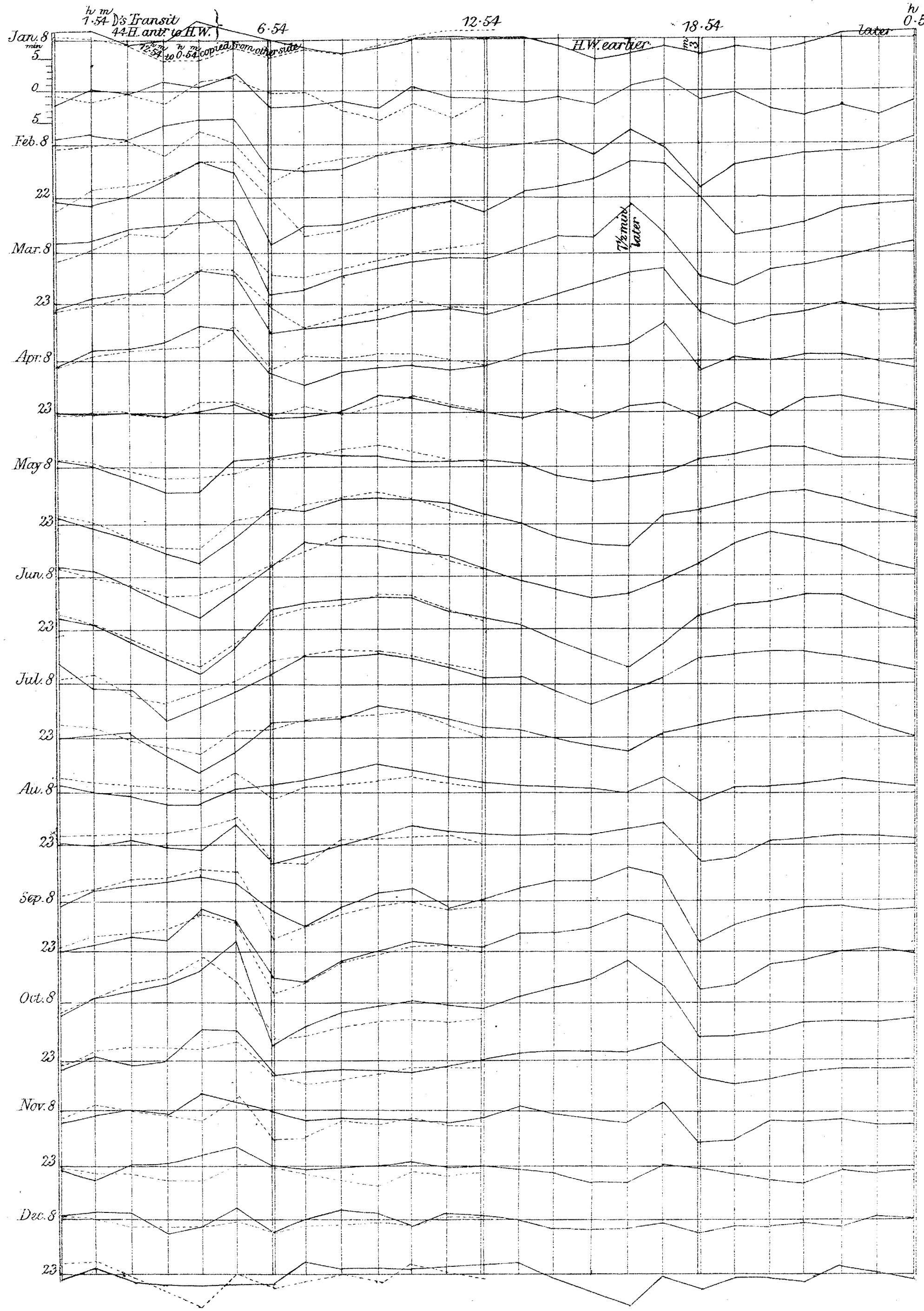


Fig. 4.

Parallax.

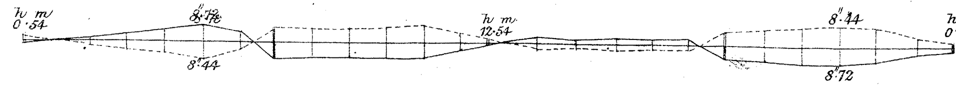
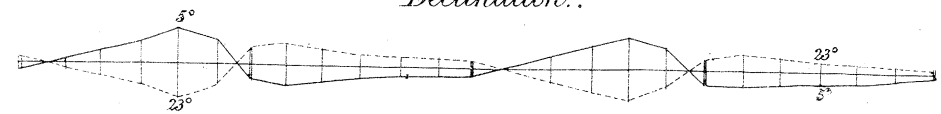


Fig. 5.

Declination.



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